

Ranking Bank Branches with Interval Data By IAHP and TOPSIS

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ABSTRACT

This paper proposes a method for ranking decision making units (DMUs) using some of the multiple criteria decision making / multiple attribute decision making (MCDM /MADM) techniques, namely, interval analytic hierarchy process (IAHP) and the technique for order preference by similarity to an ideal solution (TOPSIS). Since the efficiency score of unity is assigned to the efficient units, we determine the efficient units by standard DEA models, and calculate the weights of the criteria using IAHP. It should be mentioned that the judgments are made crisp in the interval pairwise comparison matrix by the Monte Carlo simulation. In the end, we utilize TOPSIS using IAHP to rank bank branches in Iran.

Keywords

Interval analytic hierarchy process, Monte Carlo simulation, TOPSIS, Ranking.

1. Introduction

DEA, proposed by Charnes et al. [2], is a method for evaluating DMUs. The first and second basic DEA models were proposed by Charnes et al. [2] and Banker et al. [1], respectively. In basic DEA models, efficient units receive the efficiency score of unity and inefficient ones receive scores less than one. Efficient units will finally be included in the ranking list. In basic DEA models, the data are assumed to be definite. But in different application of DEA, the data can be probabilistic, interval, or fuzzy. Thus, in this paper, the case in which the data are interval is considered. IAHP has been recently proposed by Jablonsky [4] to measure the relative efficiency of production units. TOPSIS, a multiple attribute

decision making technique developed by Hwang and Yoon [3], received a lot of attention, since it has a strong logic and considers both ideal and non-ideal solutions. Recently AHP and TOPSIS have been used in the fuzzy environment by Öñüt and Soner [5]. In order to solve a problem, TOPSIS requires the weights of the criteria. Thus, among the techniques for calculating weights, IAHP is more reliable because of having advantages such as pairwise comparison, using interval judgments rather than judgments with precise values, and computing the incompatibility rate. The interval data and interval pairwise comparison are turned into crisp data by using Monte Carlo (MC) simulation. This paper is organized as follows. The proposed model is described in section 2. The theoretic descriptions for IAHP and TOPSIS methods are presented in section 2.1 and 2.2. In section three, the results of the proposed method are elaborated on using a numerical example. Finally, the paper is concluded in section 4.

2. The propose method

In this section, IAHP and TOPSIS are introduced. As the data obtained from the data collection may not be precise in practical tasks, we consider the data as intervals; where these are turned into crisp data by using MC simulation. Moreover, in order for the judgments to be more realistic, the judgment intervals are determined by the decision maker, which can be computed with the possibility of computing the decision incompatibility rate in AHP, to make sure of the compatibility of the decision. Then, using Saaty's [6] eigenvector problem, the relative weight can be obtained. The weights of the criteria are then calculated by IAHP. Finally, TOPSIS uses the weights of the criteria obtained by IAHP and performs the final ranking of the choices by presenting

the best suggested solution. In the following subsections, IAHP and TOPSIS are reviewed.

2.1. IAHP

AHP is a multiple attribute decision making technique in which both quantitative and qualitative criteria can be used. In the hierarchy diagram, goals hold the highest position, criteria and subcriteria are placed in the next level, and choices are in the lowest position. In order to use AHP for ranking, the pairwise comparison matrix is obtained from the preferences derived from Saaty’s [6] nine-point scale; that is as follows.

Table 1: Scale of importance

Intensity of importance	Definition
1	Equal importance
3	Weak importance of one over another
5	Essential or strong importance
7	Very strong or demonstrated importance
9	Absolute importance
2, 4, 6, 8	Intermediate values between adjacent scale values

The weights derived from comparison between criteria and goals and between choices and criteria are combined to obtain the final weight. Considering the uncertainty and complexity of real-world problems, it is more realistic for the decision maker to use oral expressions rather than accurate comparisons and interval judgments. In general, interval pairwise comparison matrices is as follows.

$$A = [a_{ij}] = \begin{bmatrix} 1 & [a'_{12}, a''_{12}] & \dots & [a'_{1k}, a''_{1k}] \\ [a'_{21}, a''_{21}] & 1 & \dots & [a'_{2k}, a''_{2k}] \\ \vdots & \vdots & \ddots & \vdots \\ [a'_{k1}, a''_{k1}] & [a'_{k2}, a''_{k2}] & \dots & 1 \end{bmatrix}, i, j = 1, \dots, k \tag{1}$$

where a'_{ij} is the lower bound and a''_{ij} is the upper bound for the elements a_{ij} , and they reflect the valuation of unit i over unit j . free cells are fill by reciprocal condition as follows:

$$a'_{ij} = \frac{1}{a''_{ij}}, i, j = 1, \dots, k. \tag{2}$$

We conduct 1000 simulation experiments for transforming the interval comparisons to crisp. Then, we

calculate the weights using methods for obtaining weights, such as Saaty’s [6] eigenvector problem.

$$A \times w_i = \lambda_{\max} w_i, i = 1, \dots, k, \tag{3}$$

$$\sum_{i=1}^k w_i = 1,$$

where λ_{\max} is the largest eigenvalue of A and w is the normalized eigenvector belonging to λ_{\max} . One of the advantages of AHP is its ability to determine the compatibility of the decision.

$$IR = \frac{\lambda_{\max} - k / k - 1}{IIR}, \tag{4}$$

where in Saaty [6], IIR is called random index of matrices. Thus, as stated by Saaty [6], if $IR < 0.1$, then the compatibility of the decision is acceptable; otherwise, the decision maker had better revise the decision.

2.2. TOPSIS

TOPSIS is one of the multiple attribute decision making techniques for presenting the best suggested solution. The selected choice should have the shortest distance from the positive ideal solution and, meanwhile, the longest distance from the negative ideal solution. The attributes “profit” and “cost” are considered as the positive and negative ideal solutions, respectively. TOPSIS simultaneously examines the distance of the choice from both positive and negative ideal solutions, by calculating the relative closeness to the positive ideal solution. It also evaluates the decision matrix containing m alternatives and n attributes.

$$A = \begin{matrix} & C_1 & .. & C_2 & \dots & C_j & \dots & C_n \\ \begin{matrix} A_1 \\ A_2 \\ \vdots \\ A_i \\ \vdots \\ A_m \end{matrix} & \begin{bmatrix} x_{11} & x_{12} & \dots & x_{1j} & \dots & x_{1n} \\ x_{21} & x_{22} & \dots & x_{2j} & \dots & x_{2n} \\ \vdots & \vdots & & \vdots & & \vdots \\ x_{i1} & x_{i2} & \dots & x_{ij} & \dots & x_{in} \\ \vdots & \vdots & & \vdots & & \vdots \\ x_{m1} & x_{m2} & \dots & x_{mj} & \dots & x_{mn} \end{bmatrix} \end{matrix} \tag{5}$$

where $A_i, i = 1, \dots, m$ are the alternatives and $x_{ij}, i = 1, \dots, m, j = 1, \dots, n$ is the numerical value obtained from the i th alternative and the j th attribute. The stages of using TOPSIS are as follows

1. Normalizing the decision matrix,

$$r_{ij} = \frac{x_{ij}}{\sum_{i=1}^m x_{ij}^2}, \quad i = 1, \dots, m, j = 1, \dots, n \tag{6}$$

2. Weighting the normalized decision matrix:

$$v_{ij} = w_i \times r_{ij}, \quad i = 1, \dots, m, j = 1, \dots, n, \tag{7}$$

$$\sum_{i=1}^n w_i = 1,$$

where w_i are the weights obtained by IAHP for the attributes,

3. Determining the positive and negative ideal solutions: we define two virtual alternatives A^* and A^- as the most and the least effective choices, respectively,

$$A^* = \{(\max_i v_{ij} | j \in I), (\min_i v_{ij} | j \in J) | i = 1, \dots, k\} \tag{8}$$

Subject to:

$$I = \{j = 1, \dots, n | I \text{ is associated with benefit criteria.}\}, \tag{9}$$

$$J = \{j = 1, \dots, n | J \text{ is associated with cost criteria.}\}$$

4. Calculating the distances: the distance of each choice can be measured by the Euclidean method,

$$d_i^+ = \sqrt{\sum_{j=1}^n (v_{ij} - v_j^*)^2}, \quad i = 1, \dots, k \tag{10}$$

$$d_i^- = \sqrt{\sum_{j=1}^n (v_{ij} - v_j^-)^2}, \quad i = 1, \dots, k \tag{11}$$

5. Calculating the relative closeness of A_i^- to A_i^* ,

$$R_i = \frac{d_i^-}{d_i^+ + d_i^-}, \quad i = 1, \dots, k \tag{12}$$

6. Ranking the choices: the alternatives can be ranked in descending order of R_i .

3. Numerical example

We consider 20 bank branches with nine attributes with interval data. The first three criteria were taken as inputs and the remaining ones as outputs of the model.

1. Interest paid
2. Working hour of Personnel
3. Demand
4. Interest-free saving account
5. Current account
6. Short-term
7. Long-term
8. Loans
9. Interest received

The interval input and output of this study are given in tables 2 and 3

Table 2: Interval inputs of bank branches.

units	Interest paid	W.H. Personnel	Demand
1	[12500.8,13000.8]	[36.7,38.7]	[134000,135000]
2	[14500.6,15000.6]	[21.7,23.7]	[10000,10500]
3	[11500.02,12000.02]	[24.5,26.5]	[40000,45000]
4	[850.8,900.8]	[19.9,21.9]	[44000,44500]
5	[4500.9,5000.9]	[13.4,15.4]	[13000,13500]
6	[9000.03,9500.03]	[17.8,19.8]	[85000,90000]
7	[300.6,350.6]	[24.6,26.6]	[97500,98000]
8	[3100.7,3500.7]	[25.2,27.2]	[7500,8000]
9	[800.02,850.02]	[20.9,22.9]	[430500,440000]
10	[11500.09,12000.09]	[15.7,17.7]	[7500,8000]
11	[2500.7,3000.7]	[20.2,22.2]	[1000,1500]
12	[3500.4,4000.4]	[22.6,24.6]	[53000,53500]
13	[6000.7,6500.7]	[20.8,22.8]	[27400,27600]
14	[2500.9,3000.9]	[20.5,22.5]	[17500,18000]
15	[2500.2,3000.2]	[36.5,38.5]	[50000,50500]
16	[4000,45000]	[23.5,25.5]	[32500,33000]
17	[6000.3,6500.3]	[24.7,26.7]	[41500,42000]
18	[500.9,550.9]	[15.2,17.2]	[22000,22500]
19	[3500.9,4000.9]	[27.7,29.7]	[16500,17000]
20	[900.4,950.4]	[29.4,31.4]	[30000,30500]

Table 3: Interval outputs of bank branches.

units	saving account	Current account	Short-term	Long-term	Loans	Interest received
1	[709000,710000]	[261000,262000]	[171500,172500]	[342500,343000]	[2700000,2800000]	[58000.5,60000.5]
2	[73500,74000]	[317000,318000]	[2000,2500]	[26000,27000]	[251000,253000]	[3400.6,3600.6]
3	[435000,436000]	[489000,490000]	[6400,6600]	[78500,79500]	[1500000,2500000]	[5700.5,5900.5]
4	[29000,29500]	[971000,972000]	[1500,1600]	[45500,46500]	[452000,454000]	[500.4,700.4]
5	[216000,216500]	[158000,159000]	[800,900]	[89500,90500]	[552000,553000]	[48500.8,50500.8]
6	[514000,515000]	[508000,509000]	[102500,103500]	[69000,70000]	[1100000,2100000]	[33000.5,35000.5]
7	[15500,16000]	[172000,173000]	[750,850]	[7300,7400]	[537000,538000]	[1000.3,1200.3]
8	[155500,156000]	[269000,270000]	[5000,5500]	[83000,84000]	[1000000,2000000]	[900.3,1100.3]
9	[13700,13800]	[163000,164000]	[5000,6000]	[86000,87000]	[4300000,5300000]	[13000.6,14000.6]
10	[666600,667000]	[242000,243000]	[2300,2400]	[5100,5200]	[2500000,3000000]	[21000.8,22000.8]
11	[106500,107000]	[408000,409000]	[2400,2500]	[172500,173500]	[1400000,2400000]	[2000.8,3000.8]
12	[190000,190500]	[174000,175000]	[9350,9450]	[109500,110500]	[509000,511000]	[2000.1,3000.1]
13	[337000,337500]	[293000,294000]	[12200,12400]	[147500,148500]	[700000,800000]	[1000.4,3000.4]
14	[132800,133100]	[131000,132000]	[15000,16000]	[68500,69500]	[421000,423000]	[300.05,500.05]
15	[152500,153000]	[189000,190000]	[11000,12000]	[40500,41500]	[720000,740000]	[1200.9,3200.9]
16	[245000,246000]	[105500,106000]	[3000,4000]	[11500,12500]	[445000,450000]	[1100.5,1300.5]
17	[392000,392500]	[646000,647000]	[5500,6500]	[30000,31000]	[2000000,2200000]	[10000.3,11000.3]
18	[27500,28000]	[196000,197000]	[1500,2500]	[5500,6500]	[27000,29000]	[300.2,400.2]
19	[203800,204100]	[501500,502500]	[3000,4000]	[38500,39500]	[1500000,2500000]	[8500.1,9000.1]
20	[52000,52500]	[71000,71500]	[12500,13500]	[14500,15000]	[220000,300000]	[2200.2,3200.2]

The interval data are turned into crisp data by MC simulation, then they are normalized.

The crisp state of data in Table 2,3 are used as the inputs and outputs of the BCC model, and efficient DMUs are identified. Now we make interval judgments for 14 BCC-efficient alternatives using IAHP and we can use IAHP process in this stage .

$$\begin{matrix}
 I_1 & I_2 & I_3 & O_1 & O_2 & O_3 & O_4 & O_5 & O_6 \\
 \begin{matrix} I_1 \\ I_2 \\ I_3 \\ O_1 \\ O_2 \\ O_3 \\ O_4 \\ O_5 \\ O_6 \end{matrix} & \begin{bmatrix} 1 & [1,2] \\ & 1 \\ [2,3] & [2,3] & 1 \\ [5,7] & [6,8] & [5,7] & 1 & [5,7] & [4,6] & [4,6] \\ [6,8] & [7,9] & [6,8] & [3,5] & 1 & [6,8] & [4,6] & [4,6] \\ [2,3] & [1,2] & [2,3] & & & 1 \\ [3,5] & [2,3] & [3,5] & & [1,2] & 1 & [1,2] \\ [6,8] & [7,9] & [5,7] & [6,8] & [4,6] & [3,5] & [4,6] & 1 & [6,8] \\ [2,3] & [1,2] & [2,3] & & & [1,2] & & & 1 \end{bmatrix}
 \end{matrix}$$

Table 3: Crisp pairwise comparisons and calculations.

	I_1	I_2	I_3	O_1	O_2	O_3	O_4	O_5	O_6	w_i	$A \times w_i$	λ_{max}	IR
I_1	1	1.51	0.4	0.167	0.143	0.4	0.251	0.143	0.251	0.025	0.2371	9.484	
I_2	0.662	1	0.4	0.143	0.125	0.662	0.4	0.125	0.662	0.025	0.2442	9.768	
I_3	2.5	2.5	1	0.167	0.143	0.4	0.251	0.167	0.662	0.039	0.3511	9.003	
O_1	5.99	6.99	5.99	1	0.251	5.99	5	0.2	5	0.173	1.7312	10.007	
O_2	6.99	8.03	6.99	3.99	1	6.99	5	0.2	5	0.227	2.5535	11.249	
O_3	2.5	1.51	2.5	0.167	0.143	1	0.125	0.251	0.251	0.045	0.4115	9.144	
O_4	3.99	2.5	3.99	0.2	0.2	1.51	1	0.2	1.51	0.072	0.6804	9.450	
O_5	6.99	8.03	5.99	6.99	5	3.99	5	1	6.99	0.343	4.1784	12.182	
O_6	3.99	1.51	1.51	0.2	0.2	1.51	0.251	0.143	1	0.049	0.4605	9.398	0.0832

As stated by Saaty [6], considering the incompatibility rate of the system < 0.1 , system compatibility is acceptable; in other words, IAHP computations are compatible. As the criterion matrix has been normalized in IAHP, now we weight the normalized decision matrix using the weights obtained from IAHP, and then determine the positive and negative ideal solutions. The results obtained from calculating the distances and ranking are presented in the table 4.

Table 4: The efficiency scores and ranking.

units	θ_{BCC}^*	d_i^*	d_i^-	R_i	Rank
1	1.0000	0.1485	0.3539	0.7044	1
2	0.7554
3	0.7231
4	1.0000	0.3901	0.0954	0.1965	11
5	1.0000	0.3963	0.0881	0.1819	12
6	1.0000	0.3778	0.3216	0.4598	4
7	1.0000	0.4198	0.0529	0.1119	13
8	1.0000	0.3487	0.1215	0.2584	9
9	1.0000	0.2904	0.3595	0.5532	2
10	1.0000	0.2607	0.2704	0.5091	3
11	1.0000	0.3531	0.1497	0.2977	8
12	0.9572
13	1.0000	0.3720	0.1105	0.2290	10
14	0.9565
15	0.9497
16	0.9413
17	1.0000	0.2807	0.1890	0.4024	5
18	1.0000	0.4442	0.0463	0.0944	14
19	1.0000	0.3106	0.1594	0.3392	7
20	1.0000	0.3253	0.1958	0.3757	6

As can be seen from the above table, the first alternative holds the first rank and the 12th alternative has the last rank.

4. Conclusion

The aim of this paper was to rank bank branches using MCDM / MADM techniques, "TOPSIS" and "IAHP". It can be observed from the results of the evaluation of 20 bank branches, using IAHP and TOPSIS, that the first and the 12th branches have the best and the worst ranks, respectively, in the ranking list, as compared to the ranks of other candidates. This paper utilized TOPSIS to rank bank branches using IAHP. Since interval judgments are more assuring, more flexible and more realistic, IAHP has been used to determine the weights of the criteria in the our proposed method.

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